Homework 7

1. Find the derivative of the function

$$y = \ln \frac{x}{x^2 + 1}$$

2. Find the integral

   (a)

$$\int \frac{x(x-2)}{(x-1)^3}\ dx$$

   (b)

$$\int \frac{\sec x \tan x}{\sec x - 1} dx$$

Sol：

1.

$$y = \ln\frac{x}{x^2+1} = \ln x - \ln x^2 + 1$$

$$y' = \frac{1}{x} - \frac{2x}{x^2+1} = \frac{1-x^2}{x(x^2+1)}$$

2.

(a)

$$\int \frac{x(x-2)}{(x-1)^3}\ dx = \int \frac{x^2-2x+1-1}{(x-1)^3}\ dx$$

$$= \int \frac{(x-1)^2}{(x-1)^3}\ dx - \int \frac{1}{(x-1)^3}\ dx$$

$$= \int \frac{1}{(x-1)}\ dx - \int \frac{1}{(x-1)^3}\ dx$$

$$= \ln|x-1| + \frac{1}{2(x-1)^2} + C$$

(b)

$$u = \sec x - 1, du = \sec x \tan x\, dx$$

$$\int \frac{\sec x \tan x}{\sec x - 1} dx = \ln|\sec x - 1| + C$$